

Five-dimensional rotating black holes in Einstein-Gauss-Bonnet theory

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Abstract

We present arguments for the existence of five-dimensional rotating black holes with equal magnitude angular momenta in Einstein-Gauss-Bonnet theory with negative cosmological constant. These solutions possess a regular horizon of spherical topology and approach asymptotically an Anti-de Sitter spacetime background. We discuss the general properties of these solutions and, using an adapted counterterm prescription, we compute their entropy and conserved charges.

1 Introduction

In five dimensions, the most general theory of gravity leading to second order field equations for the metric is the so called Einstein-Gauss-Bonnet (EGB) theory, which contains quadratic powers of the curvature. The Gauss-Bonnet term appears as the first curvature stringy correction to general relativity [1, 2], when assuming that the tension of a string is large as compared to the energy scale of other variables.

The study of black holes with higher derivative curvature in Anti-de Sitter (AdS) spaces has been considered by many authors in the recent years. Static AdS black hole solutions in EGB gravity are known in closed form, presenting a number of interesting features (see *e.g.* [3], [4], [5], [6] and the references therein). It is of interest to generalize these solutions by including the effects of rotation. This problem has been considered recently in [7] within a perturbative approach. The authors of [7] discussed some properties of a particular set of asymptotically AdS_d ($d > 4$) rotating solutions in EGB theory with one nonvanishing angular momentum (where the rotation parameter appears as a small quantity), the effects of an U(1) field being also included.

The main purpose of this work is to present numerical evidence for the existence of a different class of rotating solutions in $d = 4 + 1$ EGB theory with negative cosmological constant, approaching asymptotically an AdS spacetime background. These solutions are found within a nonperturbative approach, by directly solving the EGB equations with suitable boundary conditions. They possess a regular horizon of spherical topology and have two equal magnitude angular momenta. This leads to a system of coupled nonlinear ordinary differential equations (ODEs), which are solved numerically. The same approach has been employed recently to construct Einstein-Maxwell rotating black hole solutions in higher dimensions [8, 9].

2 The general formalism

2.1 The action and boundary counterterms

We consider the EGB action with a negative cosmological constant $\Lambda = -6/\ell^2$

$$I = \frac{1}{16\pi G} \int_{\mathcal{M}} d^5x \sqrt{-g} \left(R - 2\Lambda + \frac{\alpha}{4} L_{GB} \right), \quad (2.1)$$

where R is the Ricci scalar and

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}, \quad (2.2)$$

is the Gauss-Bonnet term. The variation of the action (2.1) with respect to the metric tensor results in the equations of the model

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{\alpha}{4}H_{\mu\nu} = 0, \quad (2.3)$$

where

$$H_{\mu\nu} = 2(R_{\mu\sigma\kappa\tau}R_{\nu}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}_{\nu} + RR_{\mu\nu}) - \frac{1}{2}L_{GB}g_{\mu\nu}. \quad (2.4)$$

For a well-defined variational principle, one has to supplement the action (2.1) with the Gibbons-Hawking surface term

$$I_b^{(E)} = -\frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^4x \sqrt{-\gamma} K, \quad (2.5)$$

and its counterpart for the Gauss-Bonnet gravity [2]

$$I_b^{(GB)} = -\frac{\alpha}{16\pi G} \int_{\partial\mathcal{M}} d^4x \sqrt{-\gamma} (J - 2G_{ab}K^{ab}), \quad (2.6)$$

where γ_{ab} is the induced metric on the boundary, K is the trace of the extrinsic curvature of the boundary, G_{ab} is the Einstein tensor of the metric γ_{ab} and J is the trace of the tensor

$$J_{ab} = \frac{1}{3}(2KK_{ac}K_b^c + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}). \quad (2.7)$$

To compute the conserved charges of the asymptotically AdS solutions in EGB gravity, we use the approach proposed by Balasubramanian and Kraus in [10]. This technique was inspired by AdS/CFT correspondence and consists in adding suitable counterterms I_{ct} to the action of the theory in order to ensure the finiteness of the boundary stress tensor $T_{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta I}{\delta \gamma^{ab}}$ derived by the quasilocal energy definition [11].

Therefore we supplement the general action (which contains the surface terms for Einstein and Gauss-Bonnet gravity) with the following boundary counterterm

$$I_{ct} = \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^4x \sqrt{-\gamma} (c_1 - \frac{c_2}{2}R), \quad (2.8)$$

where R is the curvature scalar associated with the induced metric γ (see also [12] for previous work on boundary counterterm technique in EGB gravity, applied to non-rotating solutions). The consistency of the procedure fixes the expression¹ of c_1, c_2 :

$$c_1 = \frac{-1 - \frac{2\alpha}{\ell^2} + \sqrt{1 - \frac{2\alpha}{\ell^2}}}{\sqrt{\alpha} \sqrt{1 - \sqrt{1 - \frac{2\alpha}{\ell^2}}}}, \quad c_2 = \frac{\sqrt{\alpha} \left(3 - \frac{2\alpha}{\ell^2} - 3\sqrt{1 - \frac{2\alpha}{\ell^2}} \right)}{2 \left(1 - \sqrt{1 - \frac{2\alpha}{\ell^2}} \right)^{\frac{3}{2}}}. \quad (2.9)$$

Varying the total action with respect to the boundary metric γ_{ab} , we find the divergence-free boundary stress-tensor

$$T_{ab} = \frac{1}{8\pi G} \left(K_{ab} - K\gamma_{ab} + c_1\gamma_{ab} + c_2G_{ab} + \frac{\alpha}{2}(Q_{ab} - \frac{1}{3}Q\gamma_{ab}) \right), \quad (2.10)$$

where [13]

$$Q_{ab} = 2KK_{ac}K_b^c - 2K_{ac}K^{cd}K_{db} + K_{ab}(K_{cd}K^{cd} - K^2) + 2KR_{ab} + RK_{ab} - 2K^{cd}R_{cadb} - 4R_{ac}K_b^c, \quad (2.11)$$

¹ As $\alpha \rightarrow 0$, one recovers the known expression in Einstein gravity, $c_1 \rightarrow -3/\ell + \alpha/4\ell^3 + O(\alpha)^2$, $c_2 \rightarrow \ell/2 + 3\alpha/8\ell + O(\alpha)^2$.

with R_{abcd} , R_{ab} denoting the Riemann and Ricci tensors of the boundary metric.

Provided the boundary geometry has an isometry generated by a Killing vector ξ^i , a conserved charge

$$\mathfrak{Q}_\xi = \oint_\Sigma d^3 S^i \xi^j T_{ij} \quad (2.12)$$

can be associated with a closed surface Σ [10]. Physically, this means that a collection of observers on the hypersurface whose metric is γ all observe the same value of \mathfrak{Q}_ξ provided this surface has an isometry generated by ξ .

2.2 The metric ansatz and known limits

While the general EGB rotating black holes would possess two independent angular momenta and a more general topology of the event horizon, we restrict here to configurations with two equal magnitude angular momenta and a spherical horizon topology. The suitable metric ansatz² reads [8]

$$ds^2 = \frac{dr^2}{f(r)} + g(r)d\theta^2 + h(r)\sin^2\theta(d\varphi - w(r)dt)^2 + h(r)\cos^2\theta(d\psi - w(r)dt)^2 + (g(r) - h(r))\sin^2\theta\cos^2\theta(d\varphi - d\psi)^2 - b(r)dt^2 \quad (2.13)$$

where $\theta \in [0, \pi/2]$, $(\varphi, \psi) \in [0, 2\pi]$, r and t being the radial and time coordinates. For such solutions, the isometry group is enhanced from $R \times U(1)^2$ to $R \times U(2)$, where R denotes the time translation. This symmetry enhancement allows us to deal with ODEs (in what follows, we fix the metric gauge by taking $g(r) = r^2$).

For the metric ansatz (2.13), the EGB field equations (2.3) present two well known exact solutions. The first one corresponds to the generalization [3] of the static Schwarzschild-AdS solution with a Gauss-Bonnet term³

$$f(r) = b(r) = 1 + \frac{r^2}{\alpha} \left(1 - \sqrt{1 + 2\alpha \left(\frac{m}{r^4} - \frac{1}{\ell^2} \right)} \right), \quad g(r) = h(r) = r^2, \quad w(r) = 0. \quad (2.14)$$

The AdS₅ generalization [16, 17] of the Myers-Perry rotating black holes [18] with equal magnitude angular momenta is found for $\alpha = 0$ (no Gauss-Bonnet term) and has

$$f(r) = 1 + \frac{r^2}{\ell^2} - \frac{2\hat{M}\Xi}{r^2} + \frac{2\hat{M}\hat{a}^2}{r^4}, \quad h(r) = r^2 \left(1 + \frac{2\hat{M}\hat{a}^2}{r^4} \right), \quad w(r) = \frac{2\hat{M}\hat{a}}{r^2 h(r)}, \quad g(r) = r^2, \quad b(r) = \frac{r^2 f(r)}{h(r)} \quad (2.15)$$

where \hat{M} and \hat{a} are two constants related to the solution's mass-energy and angular momentum, while $\Xi = 1 - \hat{a}^2/\ell^2$.

3 Black hole properties

We are interested in black hole solutions with an horizon located at a constant value of the radial coordinate $r = r_h > 0$. Restricting to nonextremal solutions, the following expansion holds near the event horizon:

$$f(r) = f_1(r - r_h) + O(r - r_h)^2, \quad h(r) = h_h + O(r - r_h), \quad b(r) = b_1(r - r_h) + O(r - r_h)^2, \quad w(r) = w_h + O(r - r_h) \quad (3.1)$$

The event horizon parameters r_h , f_1 , b_1 , w_h and h_h (with $(f_1, b_1, h_h) > 0$) are related in a complicated way to the global charges of the solutions.

² EGB rotating topological black hole with zero scalar curvature of the event horizon are known in closed form (see *e.g.* [14], [15] and references there). However, they are found for a different metric ansatz and present rather different properties.

³ Note that the EGB gravity presents two kind of static black hole solutions, which are classified into the plus and the minus branches, $f_{(\pm)}(r) = b_{(\pm)}(r) = 1 + \frac{r^2}{\alpha} \left(1 \pm \sqrt{1 + 2\alpha \left(\frac{m}{r^4} - \frac{1}{\ell^2} \right)} \right)$. In this paper we shall restrict to the minus branch solutions, which present a well defined Einstein gravity limit.

The metric functions have the following asymptotic behaviour in terms of the constants⁴ f_2 , b_2 and w_4 :

$$\begin{aligned} f &= 1 + \frac{r^2}{\alpha} \left(1 - \sqrt{1 - \frac{2\alpha}{\ell^2}} \right) + \frac{f_2}{r^2} + O(1/r^4), \quad b = 1 + r^2 \frac{1 - \sqrt{1 - \frac{2\alpha}{\ell^2}}}{\alpha} + \frac{b_2}{r^2} + O(1/r^4), \\ h &= r^2 + \ell^2 \frac{f_2 - b_2}{2r^2} \left(1 + \sqrt{1 - \frac{2\alpha}{\ell^2}} \right) + O(1/r^6), \quad w(r) = \frac{w_4}{r^4} + O(1/r^8). \end{aligned} \quad (3.2)$$

One can see that, similar to the static case, the parameter α must obey $\alpha \leq \ell^2/2$, beyond which the theory is undefined. For these asymptotics, the effective cosmological constant is $\Lambda_{eff} = \Lambda(1 + \sqrt{1 - \frac{2\alpha}{\ell^2}})/2$.

The Killing vector $\chi = \partial/\partial t + \Omega_\varphi \partial/\partial \varphi + \Omega_\psi \partial/\partial \psi$ is orthogonal to and null on the horizon. For the solutions within the ansatz (2.13), the event horizon's angular velocities are all equal, $\Omega_\psi = \Omega_\varphi = \omega_h$. The Hawking temperature as found by computing the surface gravity is

$$T_H = \frac{\sqrt{b_1 f_1}}{4\pi}. \quad (3.3)$$

Another quantity of interest is the area A_H of the rotating black hole horizon

$$A_H = \sqrt{h_h} r_h^2 V_3, \quad (3.4)$$

where $V_3 = 2\pi^2$ denotes the area of the unit three dimensional sphere.

These rotating solutions present also an ergoregion inside of which the observers cannot remain stationary, and will move in the direction of rotation. The ergoregion is the region bounded by the event horizon, located at $r = r_h$ and the stationary limit surface, or the ergosurface. The Killing vector $\partial/\partial t$ becomes null on the ergosurface, *i.e.* $g_{tt} = -b(r) + h(r)w(r)^2 = 0$. The ergosurface does not intersect the horizon.

3.1 The global charges and entropy of solutions

The global charges of these solutions are computed by using the counterterm formalism⁵ presented in Section 2. The computation of the boundary stress-tensor T_{ab} is straightforward and we find the following expression for the components of interest here

$$\begin{aligned} T_\varphi^t &= \frac{1}{8\pi G} \sqrt{\frac{\alpha(1 - \frac{2\alpha}{\ell^2})}{1 - \sqrt{1 - \frac{2\alpha}{\ell^2}}}} \frac{2w_4 \sin^2 \theta}{r^4} + O(1/r^6), \quad T_\psi^t = \frac{1}{8\pi G} \sqrt{\frac{\alpha(1 - \frac{2\alpha}{\ell^2})}{1 - \sqrt{1 - \frac{2\alpha}{\ell^2}}}} \frac{2w_4 \cos^2 \theta}{r^4} + O(1/r^6), \\ T_t^t &= -\frac{1}{8\pi G} \frac{\sqrt{\alpha}}{8(1 - \sqrt{1 - \frac{2\alpha}{\ell^2}})^{3/2}} \left(3\alpha(-2 + 3\sqrt{1 - \frac{2\alpha}{\ell^2}}) + 4(-1 + \frac{2\alpha}{\ell^2} + \sqrt{1 - \frac{2\alpha}{\ell^2}})(f_2 - 4b_2) \right) \frac{1}{r^4} + O(1/r^6). \end{aligned} \quad (3.5)$$

The mass-energy E of solutions is the charge associated with the Killing vector $\partial/\partial t$,

$$\begin{aligned} E &= E^{(0)} + E^{(c)}, \quad \text{where} \\ E^{(0)} &= \frac{V_3}{8\pi G} \frac{(f_2 - 4b_2)}{2} \sqrt{1 - \frac{2\alpha}{\ell^2}}, \quad E^{(c)} = \frac{V_3}{8\pi G} \frac{3\ell^2}{16} \left(1 - \frac{6\alpha}{\ell^2} + \sqrt{1 - \frac{2\alpha}{\ell^2}} \right), \end{aligned} \quad (3.6)$$

where $E^{(c)}$ represents the Casimir term [10] in EGB gravity, presenting a nontrivial α dependence⁶ (this term appears also in the static limit (2.14)). These black holes have also two equal magnitude angular momenta $J_\varphi = J_\psi = J$, with

$$J = \frac{V_3}{8\pi G} w_4 \sqrt{1 - \frac{2\alpha}{\ell^2}}, \quad (3.7)$$

⁴The MPAdS₅ solution (2.15) has $f_2 = 2\hat{M}(\hat{a}^2/\ell^2 - 1)$, $b_2 = -2\hat{M}$, $w_4 = 2\hat{M}\hat{a}$. For the static Schwarzschild-AdS-Gauss-Bonnet solution (2.14) one finds $f_2 = b_2 = -m/\sqrt{1 - \frac{2\alpha}{\ell^2}}$, $w_4 = 0$.

⁵A different computation of the mass and angular momentum of EGB solutions was also reported namely in [19], [20], [21].

⁶In the small α limit, one finds $E^{(c)} = 3\pi\ell^2/32G - 21\alpha\pi/64G + O(\alpha)^2$. Note that the first order correction to the mass of pure global AdS₅ does not depend on the value of the cosmological constant.

representing the charges associated with the Killing vectors $\partial/\partial\varphi$, $\partial/\partial\psi$ as computed according to (2.12).

Also, in what follows it is important to use the observation that one can write

$$\frac{1}{\sin^2\theta}(R_\varphi^t + \frac{\alpha}{4}H_\varphi^t) = \frac{1}{\cos^2\theta}(R_\psi^t + \frac{\alpha}{4}H_\psi^t) = \frac{1}{2r^2}\sqrt{\frac{f}{bh}}\frac{d}{dr}\left(\sqrt{\frac{fh}{b}}hw'\left(-r^2 + \alpha(f - 4 + \frac{3h}{r^2})\right)\right), \quad (3.8)$$

$$R_t^t + \frac{\alpha}{4}(H_t^t + \frac{1}{2}L_{GB}) = \frac{1}{2r^2}\sqrt{\frac{f}{bh}}\frac{d}{dr}\left(\sqrt{\frac{fh}{b}}\left[r^2(hww' - b') + \alpha\left((f - 4 + \frac{h}{r^2} + \frac{rfh'}{h})b' + (4 - \frac{3h}{r^2} - f)hww' + rfhw'^2\right)\right]\right).$$

The gravitational thermodynamics of the EGB black holes can be formulated via the path integral approach [22, 23]. However, while the static vacuum Lorentzian solutions (2.14) extremize also the Euclidean action as the analytic continuation in time has no effect at the level of the equations of motion, this is not the case of the rotating configurations discussed in this paper. In this case it is not possible to find directly real solutions on the Euclidean section by Wick rotating $t \rightarrow i\tau$ the Lorentzian configurations⁷. In view of this difficulty one has to resort to an alternative, quasi-Euclidean approach as described in [24]. The idea is to regard the action I used in the computation of the partition function as a functional over complex metrics that are obtained from the real, stationary, Lorentzian metrics by using a transformation that mimics the effect of the Wick rotation $t \rightarrow i\tau$. In this approach, the values of the extensive variables of the complex metric that extremize the path integral are the same as the values of these variables corresponding to the initial Lorentzian metric⁸.

When computing the classical bulk action evaluated on the equations of motion, one replaces the $R - 2\Lambda + \frac{\alpha}{4}L_{GB}$ volume term with $2(R_t^t + \frac{\alpha}{4}H_t^t)$ and make use of (3.8) to express it as a difference of two boundary integrals. A straightforward calculation using the asymptotic expansion (3.2) shows that the divergencies of the boundary integral at infinity, together with the contributions from $I_b^{(E)}$ and $I_b^{(GB)}$, are regularized by I_{ct} . As a result, by using also the first set of relations in (3.8), one finds the finite expression of the classical action

$$I_{cl} = \frac{V_3}{4G}\left(\frac{1}{\sqrt{f_1b_1}}\left[(f_2 - 4b_2)\sqrt{1 - \frac{2\alpha}{\ell^2}} + \frac{3\ell^2}{8}\left(1 - \frac{6\alpha}{\ell^2} + \sqrt{1 - \frac{2\alpha}{\ell^2}}\right)\right] - \sqrt{h_h}(r_h^2 + \alpha(4 - \frac{4h_h}{r_h^2})) - \frac{4}{\sqrt{f_1b_1}}w_hw_4\sqrt{1 - \frac{2\alpha}{\ell^2}}\right). \quad (3.9)$$

Upon application of the Gibbs-Duhem relation to the partition function, one finds the entropy $S = \beta(E - 2\omega_h J) - I_{cl}$, which is the sum of one quarter of the event horizon area plus a Gauss-Bonnet correction

$$S = S_0 + S_{GB}, \quad \text{with } S_0 = \frac{V_3}{4G}r_h^2\sqrt{h_h}, \quad S_{GB} = \alpha\frac{V_3}{4G}\sqrt{h_h}\left(4 - \frac{h_h}{r_h^2}\right). \quad (3.10)$$

In the static limit, the known expression $S = \frac{V_3}{4G}(r_h^3 + 3\alpha r_h)$ is recovered, while the entropy of the MPAdS₅ solutions is $S = V_3A_H/4G$.

4 Numerical results

The EGB equations (2.3) lead to a system of four coupled second order ODEs for the metric functions⁹ $f(r)$, $b(r)$, $h(r)$ and $w(r)$. We are interested in solutions of these equations presenting the asymptotic expansion (3.1), (3.2).

⁷ Even if one could accompany the Wick rotation with various other analytical continuations of the parameters describing the solution (e.g. $\hat{a} \rightarrow i\hat{a}$ for a MPAdS₅ black hole), given the numerical nature of the configurations in this paper, there is no assurance that the modified metric functions will also be solutions of the field equations in Euclidean signature. Instead, one has to solve directly the EGB field equations for a metric ansatz with Euclidean signature.

⁸Note also that not all closed form solutions with Lorentzian signature present reasonable Euclidean counterparts, in which case one is forced again to consider a 'quasi-Euclidean' approach. The $d = 5$ asymptotically flat rotating black ring solutions provides an interesting example in this sense [25].

⁹These equations are extremely complicated (each of them containing around fifty terms) and we shall not present them here.

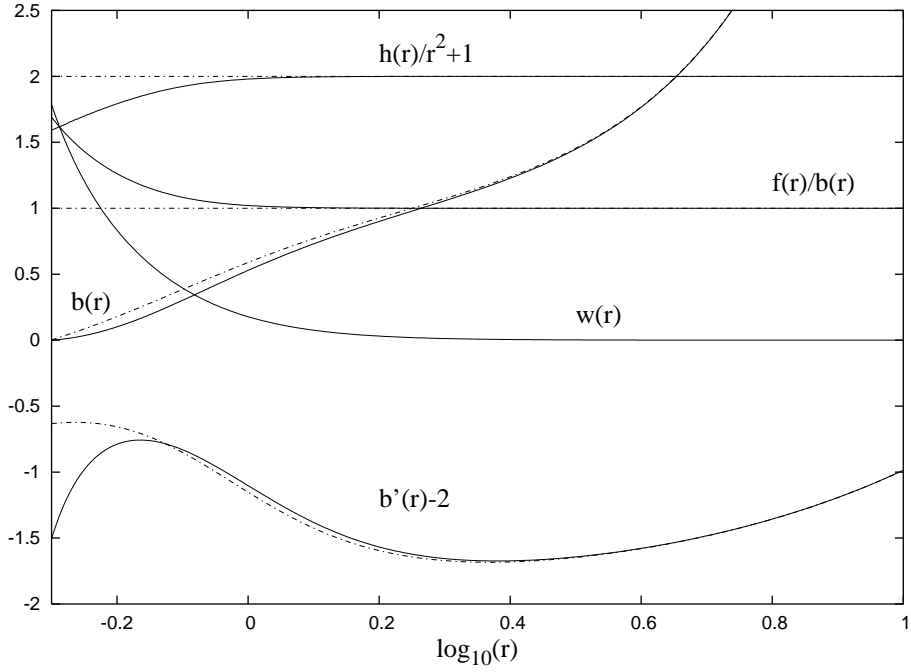


Figure 1. A typical rotating solution with $w_h = 1.8$ (solid line) is plotted together with a static solution ($w_h = 0$, dashed line) for $r_h = 0.5$, $\alpha = 0.5$, $\ell^2 = 20$.

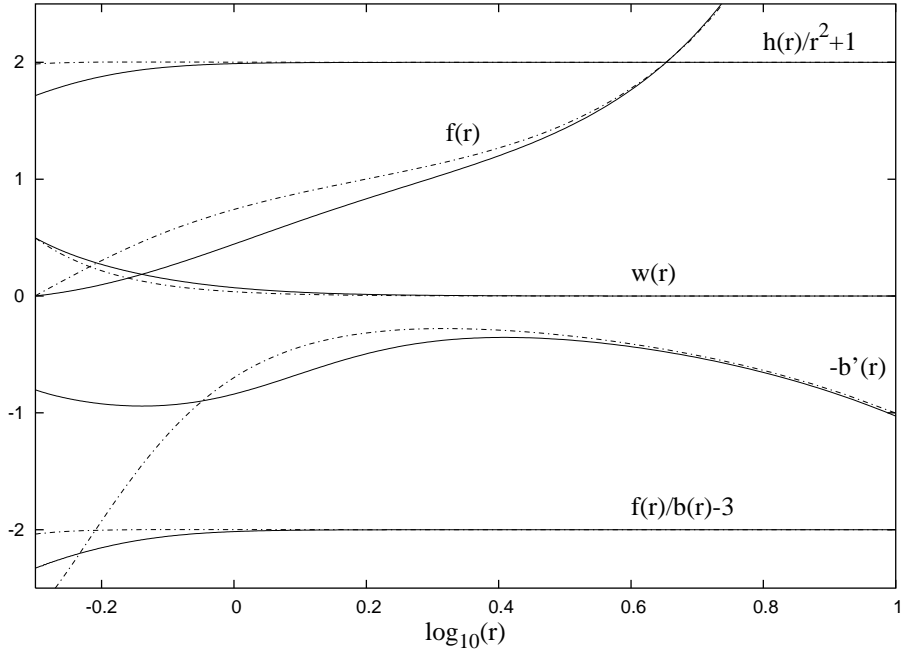


Figure 2. Two rotating solutions with $r_h = 0.5$, $w_h = 0.5$, $\ell^2 = 20$ are plotted for $\alpha = 0.1$ (dashed line) and $\alpha = 1$ (solid line).

In order to construct numerical solutions, the constants (α, Λ) have to be fixed. Then the solution is further specified by the event horizon r_h and the angular velocity at the horizon $w(r_h)$ (or equivalently, the angular momentum J through the parameter w_4).

The complete classification of the solutions in the space of parameters is a considerable task that is not aimed in this paper. Instead, by taking the arbitrary value $\ell^2 = 20$, we analyzed in detail a few particular

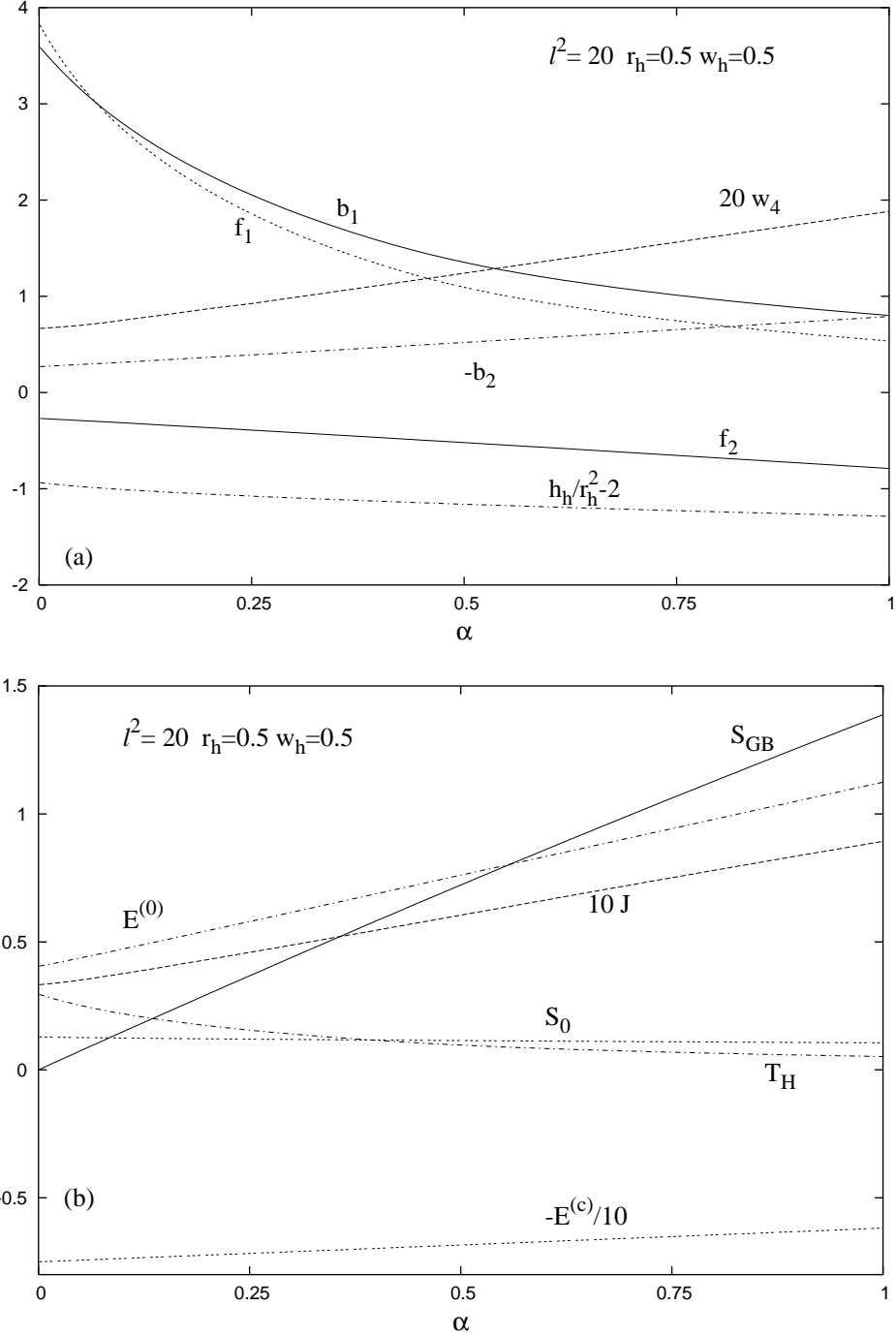


Figure 3. The parameters f_1, b_1, h_h at the event horizon are plotted together with the parameters f_2, b_2, w_4 in the asymptotic expansion at infinity, as a function of α for solutions with $r_h = 0.5$, $\ell^2 = 20$, $w_h = 0.5$ (Figure 3a). In Figure 3b we plot the Hawking temperature, the mass-energy $E^{(0)}$, the Casimir term $E^{(c)}$, the angular momentum J and the entropies S_0, S_{GB} for these solutions. Here and in Figure 4b $E^{(0)}$, $E^{(c)}$ and J are plotted in units with $V_3/8\pi G = 1$, while we set $V_3/4G = 1$ in the expression of S .

classes of solutions, which hopefully would reflect all relevant properties of the general pattern. However, we have found nontrivial rotating black hole solutions for other values of the cosmological constant, in particular for $\Lambda = 0$ and for $\Lambda > 0$.

Also, since the Gauss-Bonnet term in (2.1) has to be considered as a correction to the Einstein-Hilbert action, we report here the results for positive values of α in the interval $0 \leq \alpha \leq 1$ (however, solutions with $\ell^2 = 20$ and larger values of α exist as well).

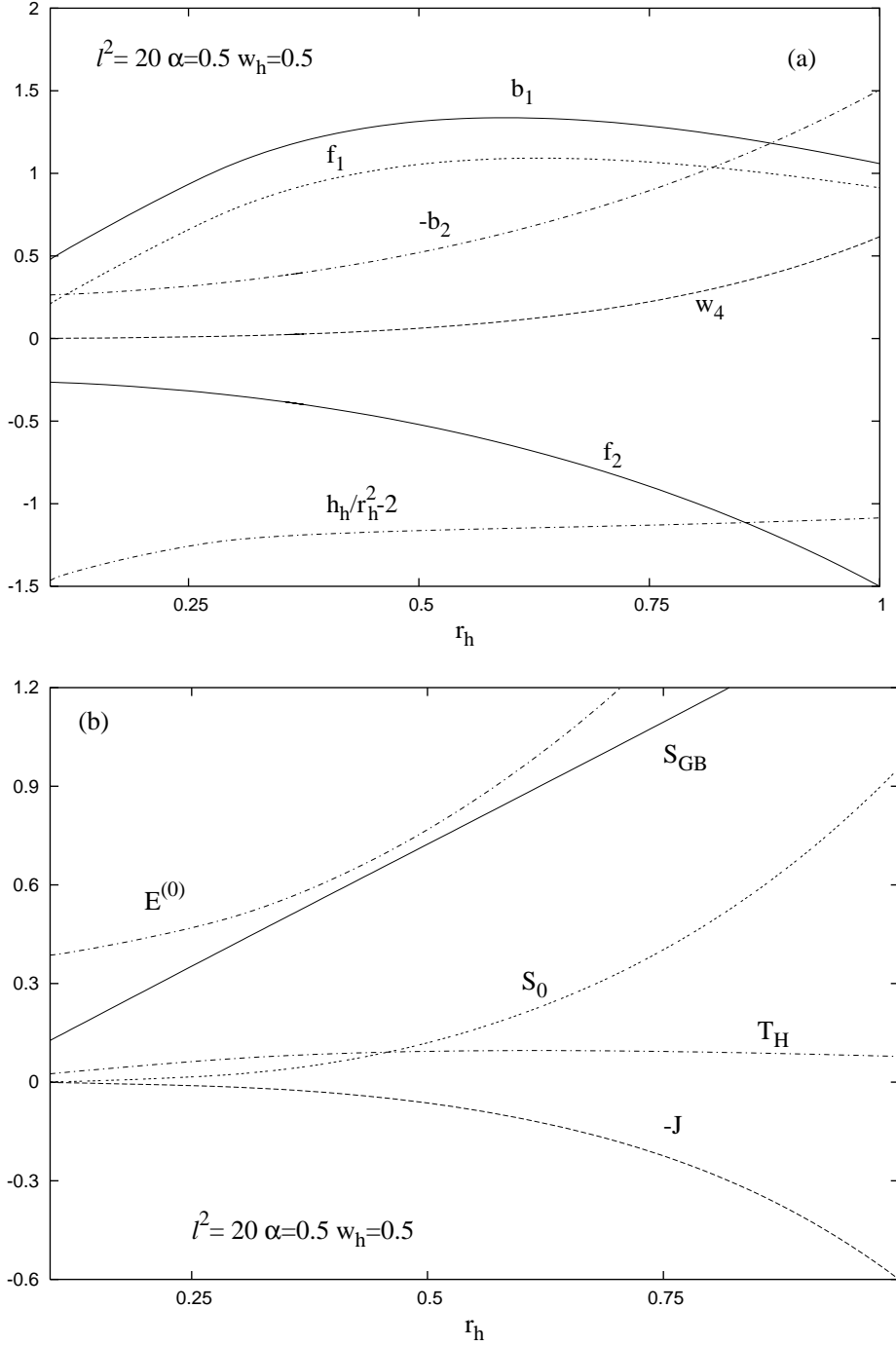


Figure 4. The parameters f_1, b_1, h_h at the event horizon are plotted together with the parameters f_2, b_2, w_4 in the asymptotic expansion at infinity, as a function of the event horizon radius r_h for solutions with $\alpha = 0.5$, $\ell^2 = 20$, $w_h = 0.5$ (Figure 4a). In Figure 4b we plot the Hawking temperature, the mass-energy $E^{(0)}$, the angular momentum J , the entropy of the solution in Einstein gravity S_0 and the Gauss-Bonnet correction S_{GB} for these solutions.

In the absence of a closed form solution, we relied on numerical methods to solve the equations. The numerical methods here are similar to those used in literature to find other numerical black hole solutions with equal magnitude angular momenta [8, 9]. We take units such that $G = 1$, and employ a collocation method for boundary-value ordinary differential equations, equipped with an adaptive mesh selection procedure [26]. Typical mesh sizes include $10^3 - 10^4$ points. The solutions have a typical relative accuracy of 10^{-8} .

In constructing rotating EGB-AdS black holes, we make use of the existence of the closed form solutions (2.14) and (2.15), and employ them as starting configurations, increasing gradually w_h or α , respectively.

The profiles of the metric functions of a typical EGB-AdS black hole solution corresponding to $\alpha = 0.5$, $r_h = 0.5$ are presented on Figure 1 for a static ($w_h = 0$) and a rotating solution with $w_h = 1.8$. One can see that the rotation leads to non constant values for $h(r)/r^2$ and $b(r)/f(r)$, and is particularly apparent on the function $b(r)$ and its derivative.

It is also natural to study how the profile of a rotating solution (*e.g.* with a given angular velocity w_h) is affected by the Gauss-Bonnet term. This is illustrated on Figure 2 where the profiles corresponding to $\alpha = 0.1$ (dashed curves) –very close to the MPAdS₅ solution, and $\alpha = 1$ (solid lines) are superposed for $r_h = 0.5$, $w_h = 0.5$. One can see there that the r^2 –terms start dominating the profile of the metric functions f, b, h very rapidly, which implies a small difference between different solutions for large enough r . However, the situation is different in the small– r regime (see also Figure 1).

We also performed an analysis of the EGB-AdS solutions when varying the Gauss-Bonnet coupling constant α . In the limit $\alpha = 0$, the MPAdS₅ black holes (2.15) are recovered¹⁰. The evolution of the parameters $f'(r_h), b'(r_h), h(r_h)$ and f_2, b_2, w_4 characterizing the solutions is shown on Figure 3a as function of α . The corresponding physical quantities, as computed according to the relations in the previous Section, are reported on Figure 3b.

We also varied the event horizon radius r_h for a set of given α , w_h and found no evidence of a maximal value of r_h where the solutions could eventually terminate. The evolution of the solution data as a function of the event horizon radius is reported on Figure 4a, the mass-energy, angular momentum, entropy and Hawking temperature being plotted in Figure 4b. For small values of r_h the numerical analysis is quite tedious and it strongly suggests that the derivatives of $w(r)$ and $h(r)/r^2$ become infinite in the $r_h \rightarrow 0$ limit.

Finally, although the numerics is more involved in this case, we constructed solutions with fixed α and r_h but varying the horizon velocity w_h . Equivalently, this leads to a family of solution with varying the angular momentum $J \sim w_4$ since there is a one-one correspondence between w_4 and w_h . Similar to the $\alpha = 0$ case, for each set of solutions we observe two branches, extending up to a maximal value of w_h , where they merge and end. The lower branch emerges from the static solution in the limit $w_h = 0$. The maximal value of w_h depends on the horizon radius r_h , the cosmological constant Λ , and the coupling constant α .

5 Further remarks

The main purpose of this paper was to present arguments for the existence of rotating black holes in $d = 4 + 1$ EGB theory with negative cosmological constant. These configurations possess a regular horizon of spherical topology and have two equal-magnitude angular momenta, representing generalizations of a particular class of MPAdS₅ black holes. We also proposed to adapt the boundary counterterm formalism of [10] to $d = 4 + 1$ EGB- Λ theory, computing in this way the mass-energy and angular momenta of solutions. The general relations in Section 2.1 apply also to other known solutions in EGB theory with negative cosmological constant and can easily be generalised for a positive sign of Λ .

The solutions in this paper may provide a fertile ground for further study of rotating configurations in EGB theory. For example, their generalization to include the effects of an electromagnetic field is straightforward. Also, in principle, by using the same techniques, there should be no difficulty to construct similar solutions in $d = 2N + 1$ dimensions with $N > 2$ equal magnitude angular momenta. An interesting problem here is to find the boundary counterterm expression in EGB– Λ theory for other values of $d > 5$.

In the five dimensional case, one can also approach the general case of a black hole with two distinct angular momenta, by solving a set of partial differential equations with a dependence on (r, θ) . The formalism proposed in Section 2 to compute the mass, angular momentum and entropy of AdS solutions should apply in the general case, too. Rotating topological black holes in EGB theory with an horizon of negative curvature are also likely to exist for $\Lambda < 0$.

The study of the solutions discussed in this paper in an AdS/CFT context is an interesting open question. According to the AdS/CFT correspondence, the higher derivatives curvature terms can be viewed as the corrections of large N expansion of the boundary CFT in the strong coupling limit. For the ansatz considered here, the boundary metric is not rotating and corresponds to a static Einstein universe in four dimensions.

¹⁰With the particular values $r_h = 0.5$, $w_h = 0.5$ that we have chosen to perform the numerical analysis, the parameters \hat{a}, \hat{M} of the MPAdS₅ solution correspond to $\hat{a} = 10/81$ and $\hat{M} = 6561/48640$.

Here it is interesting to note that, similar to the $\alpha = 0$ case, the stress-energy tensor for the dual theory defined in that background, as computed according to the standard prescription [27], is traceless.

A detailed study of the $d = 4 + 1$ rotating black hole solutions in EGB theory together with a discussion of their asymptotically flat limit will be presented elsewhere.

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Note added: When this work has been at final stages, the authors of [28] appear to have succeeded in constructing a Kerr black hole in EGB theory in $d = 4 + 1$ dimensions. However, this result needs an independant confirmation. Afterwards, it would be interesting to extend the results in Section III of [28] to the case of black holes with negative cosmological constant and two equal magnitude angular momenta.

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